1212-16-505-001

Code No. 3361 / CORE

FACULTY OF SCIENCE

M.Sc. IV-Semester Examination, May / June 2018

Subject : MATHEMATICS

Paper - I

Advanced Complex Analysis

Time : 3 hours **Max. Marks : 80**

Note : Answer all questions from Part-A and Part-B. Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8 x 4 = 32 Marks)

1 If $z_1, z_2, \ldots, z_n$ are the zeros of f inside the disc D_R than prove that

$$\int_0^R n(r)\frac{dr}{r} = \sum_{k=1}^{N}\log\left|\frac{R}{z_k}\right|$$

2 Find the growth order of $\sin \pi z$.

3 Prove that the Gamma function extends to an analytic function in the half plane $Re(s) > 0$.

4 For $n \in IN$, prove that $Res_{s=-n}\,\Gamma(s) = \frac{(-1)^n}{n!}$.

5 Prove that $(\zeta(s))^2 = \sum_{n=1}^{\infty}\frac{d(n)}{n^s}$

6 If $Re(s) > 1$, prove that $\log\,\zeta(s) = \sum_{p,m}\frac{p^{-ms}}{m}$, where p is prime, $m \in IN$.

7 For $M \in SL_2(IR)$, prove that f_M maps H onto itself where H is the upper half plane.

8 Prove that $\psi_\alpha(z) = \frac{\alpha - z}{1-\bar{\alpha}z}$, where $\alpha \in \mathbb{C}$, $|\alpha|<1$ is an automorphism of the unit disc D.

PART – B (4 x 12 = 48 Marks)

9 a) Find the Hadomards products for
i) $e^z - 1$ ii) $\sin \pi z$

OR

b) State and prove Jensen's formula.

10 a) Prove that $\lim_{n\to\infty}\frac{n^s\,n!}{s(s+1)\ldots..(s+n)} = \Gamma(s)$ for $s \neq 0, -1, -2, \ldots.$

OR

b) Prove that $\Gamma(s)\Gamma\left(s+\frac{1}{2}\right) = \sqrt{\pi}\;2^{1-2s}\,\Gamma(2s)$.

11 a) Prove that, if $\psi_1 \sim \frac{x^2}{x}$ as $x \to \infty$, then prove that $\psi(x) \sim x$ as $x \to \infty$.

OR

b) Show that the function $\xi(s) = \pi^{-\frac{s}{2}} \Gamma\left(\frac{s}{2}\right) \xi(s)$ is real when s is real or when $\text{Re}(s) = \frac{1}{2}$.

12 a) State and prove Schwarz's lemma.

OR

b) Prove that every antomorphism of upper half plane H takes the form f_M for some $M \in SL_2(\text{IR})$.

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1212-16-505-001

Code No. 3363 / CORE

FACULTY OF SCIENCE

M.Sc. IV – Semester Examination, May / June 2018

Subject: Mathematics

Paper – II

General Measure Theory

Time: 3 Hours

Max.Marks: 80

Note: Answer all questions from Part-A and Part-B.
Each question carries 4 marks in Part-A and 12 marks in Part-B.

PART – A (8x4 = 32 Marks)
[Short Answer Type]

1 Define a measure μ on a measurable space (x, β). Prove that μ is countably sub additive.

2 State and prove Monotone convergence theorem.

3 Prove that countable union of positive sets ia a positive set.

4 Suppose (x, β, μ) is a measure space and f is an integrable function on X w.r.t. μ. Prove that ν defined on β by $\nu(E) = \int_E f d\mu$ is a signed measure on β.

5 Define a μ^*-measurable set E. Suppose. $\mu^*(E) = 0$, prove that E is a μ^*-measurable set.

6 Suppose $E \subset x \, xy$ and $x \in x$. Define x – cross section of E with usual notations. Prove that

i) $\psi_{E_x}(y) = \psi_E(x, y) \; \forall \; y \in y$

ii) $\overline{E}_x = (\overline{E})_x$

7 Suppose μ^* and μ_* are the outer and inner measures induced by a measure μ on an algebra A of subsets of X. Prove that $\mu_*(E) \le \mu^*(E) \; \forall \; E \in P(x)$.

8 If A ∊ A prove that

$\mu(A) = \mu_*(A \cap E) + \mu^*(A \cap \overline{E})$.

PART – B (4x12 = 48 Marks)
[Essay Answer Type]

9 a) Suppose(x, β, μ) is a measure space. Prove that is can be extended to a complete measure space (x_1, β_0, μ_0) where $\beta \subset \beta_0$ and restriction μ_0 to β is μ i.e. $\mu_0 / \beta = \mu$.

..2

OR

b) Suppose (x, β) is a measurable space and $E \in \beta$. Suppose $f: E \to [-\infty, \infty]$ is a mapping. Prove that the following are equivalent.

i) $\{x \in E: f(x) > \alpha\} \in \beta$ for all $\alpha \in R$

ii) $\{x \in E: f(x) \geq \alpha\} \in \beta$ for all $\alpha \in R$

iii) $\{x \in E: f(x) < \alpha\} \in \beta$ for all $\alpha \in R$

iv) $\{x \in E: f(x) \leq \alpha\} \in \beta$ for all $\alpha \in R$

10 a) Suppose E is a measurable set such that $0 < \nu(E) < \infty$. Prove that E has a positive set A such that $\nu < A_1 > 0$.

OR

b) State and prove Jordon – decomposition theorem.

11 a) Prove that the class β of all μ^* - measurable sets is a σ - algebra of sets.

OR

b) Suppose (x, A, μ) and (y, β, ν) are complete measure spaces and R is the class of all measurable rectangles in X x Y. Suppose $E \in R_{\sigma\delta}$ with $(\mu \times \nu)(E) < \infty$. Prove that

i) The function $g: x \to [0, \infty]$ defined by $g(x) = \nu(E_x)$ $\forall x \in X$ is a measurable function on x. Also

ii) $\int_X g\, d\mu = \int_X \nu(E_x) d\mu = (\mu \times \nu)(E)$.

12 a) Suppose $E \subset x$ with $\mu^*(E) < \infty$. Prove that E is μ^* - measurable if and only if $\mu^*(E) = \mu_*(E)$.

OR

b) State and prove Coratheodory outer measure theorem.

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1212-16-505-001

Code No. 3381/CORE

FACULTY OF SCIENCE

M.Sc. IV – Semester Examination, May / June 2018

Subject: Mathematics / Applied Mathematics

Paper – III (A): Integral Equations and Calculus of Variations

Time : 3 Hours **Max. Marks: 80**

Note : Answer all questions from Part–A and Part–B. Each question carries 4 marks in Part–A and 12 marks in Part – B.

PART – A (8 x 4 = 32 Marks)
(Short Answer Type)

1 Find the resolvent Kernel of the Volterra integral equation with Kernel $K(x, t) = e^{x-t}$.

2 Show that $\beta(p, q) = \beta(p+1, q)+\beta(p, q, + 1)$.

3 Show that the homogeneous Integral equation

$\phi(x) = \lambda \int_0^1 (3x-2)t\varphi(t)dt = 0$ has no characteristic numbers and eigen functions.

4 Define Green's function and find Green's function for the boundary – value problem $y'' - y = 0$; $y(0)=y(1)=0$.

5 On what curves can the functional $v[y(x)] = \int_0^{\pi/2} [(y')^2 - y^2]\,dx$; $y(0) = y(\pi/2) = 1$, be extremized.

6 Find the extremal of the functional

$$v[y(x)] = \int_{x_0}^{x_1} (zxy + y'''^2)\,dx$$

7 Write the ostrogradsky equation for the functional

$$v = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 2zf(x, y)\right]dxdy$$

8 State and prove Hamilton's principle.

PART – B (4 x 12 = 48 Marks)
(Essay Answer Type)

9 (a) With aid of resolvent Kernel, find the solution of the integral equation

$$\varphi(x) = x3^x - \int_0^x 3^{x-t}\varphi(t)dt$$

OR

(b) Solve the integro-differential equation.

$$\varphi''(x) + \int_0^x e^{2(x-t)}\varphi'(t)dt = e^{2x} \; ; \varphi(0) = 0, \varphi'(0) = 1$$

10 (a) Solve the integral equation

$$\varphi(x) - \lambda \int_{-\pi}^{\pi} (x\cos t + t^2 \sin x + \cos x \sin t)\varphi(t)dt = x$$

OR

(b) Using the Green's function, solve to boundary value problem
$y'' = y = x^2$; $y(0) = y(\pi/2)=0$

11 (a) Show that the externals of the functional $t[y(x)] = \int_{x_0}^{x_1} \frac{\sqrt{1+y^2}}{x} dx$ are family of circles

OR

(b) Define minimum-surface of revolution problem and solve it.

12 (a) Derive the Euler-Poisson equation for the functional dependent on higher-order-derivatives.

OR

(b) Derive the differential equation of the free vibration of a string using the varitional principle.

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1212-16-505-001

Code No: 3372/CORE

FACULTY OF SCIENCE

M.Sc. IV Semester Examination, 2018

Subject: Mathematics

Paper – IV (C) Advanced Operation Research

Time: 3 Hours **Max. Marks: 80**

Note: Answer all questions from Part A and Part B. Each question carries 4 marks in Part – A and 12 marks in Part – B.

PART – A (8x4=32 Marks)
(Short Answer Type)

1. Explain Maximin and MiniMax principles.
2. Define a saddle point. Explain with an example.
3. Explain the terms Salvage value and shortage costs.
4. Explain ABC analysis.
5. Define a General Non-linear programming problem.
6. Obtain the necessary conditions for the non-linear programming problem: Maximize $Z = x_1^2+3x_2^2+5x_3^2$ STC.$x_1+x_2+3x_3 = 2$, $5x_1+2x_2+x_3 = 5$; x_1, x_2, x_3 0
7. Define a General Quadratic Programming Problem.
8. State the applications of Non-linear programming problem.

PART – B (4x12=48 Marks)
(Essay Answer Type)

9. (a) The pay off matrix of a game is given below. Find the best strategy for each player and the value of a play of the game to A and B.

		Player B				
		B_1	B_2	B_3	B_4	B_5
Player A	A_1	9	3	1	8	0
	A_2	6	5	4	6	7
	A_3	2	4	3	3	8
	A_4	5	6	2	2	1

(b) Show that the value of the game is $V = \frac{a_{11}a_{22}-a_{21}a_{12}}{{}_1+a_{22})-(a_{12}+a_{21})}$ for any 2x2 game without saddle point having pay off matrix from AtoB is $((a_{ij}))_{2x2}$.

OR

c) Solve the following 5x2 game graphically:

		B_1	B_2
Player A	A_1	-2	5
	A_2	-5	3
	A_3	0	-2
	A_4	-3	0
	A_5	1	-4

Contd…2…

10.(a)(i) A commodity is to be supplied at a constant rate of 200 unit per day. Supplies of any amounts can be had at any required time, but each ordering costs Rs.50, costs of holding the commodity in inventory is Rs.2/- per unit per day while the delay in the supply of the item induces a penalty of Rs.10 per unit per delay of 1 day. Find the optimal policy(q,t) where t is the re-order cycle period and q is the inventory level after re-order. What would be the best, if penalty cost becomes ?

(ii) Explain the E.O.Q system of ordering.

OR

.10.(a) Explain the steps involved in the procedure of ABC analysis.

(b) A Company purchases three items, A,B and C Their annual demand and unit prices are given in the following table.

Item	Annual Demand	Unit Price
A	1,00.000	3
B	80,000	2
C	600	96

If the company wants to place forty orders per year for all three items. What is the optimal number orders for each item?

11.(a) Solve the following non-linear programming problem graphically.

Minimize $Z = (x_1-2)^2 + (x_2-1)^2$

Subject to the constraints:

$-x_1^2+x_2 \quad 0$

$-x_1-x_2+2 \quad 0$

$x_1, x_2 \quad 0$

b) Use Kuhn-Tucker conditions to solve the non linear programming problem.

Maximize $z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$

Subject to the constraints

$x_1+x_2 \leq 2$

$2x_1+3x_2 \leq 12$

$x_1, x_2 \geq 0$

12 a) Use wolfs method to solve the QPP

Minimize $Z = x_1^2+x_2^2+x_3^2$

Subject to the constraints

$2x_1+x_2-x_3 \leq 0$

$x_1 \geq 1$

$x_2 \geq 1$

OR

b) Use Beal's method to solve the QPP

Maximize $Z = 2x_1+3x_2-x_1^2$

Subject to the constraints

$x_1 + 4x_2 \leq 4$

$x_2 + x_2 \leq 2$

$x_1 , x_2 \geq 0$

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